QUIZ 3 - CALCULUS 3 (2021/4/8)

1. (a) (6 pts) Evaluate $\int_0^1 \int_0^{x^2} xe^{x^2+y} dydx$.

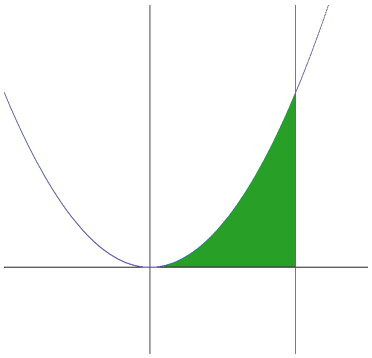
   (b) (4 pts) Sketch the region of the integration. Write down the iterated integral corresponds to the same integration, but reverse the order of the integration. **You do not need to compute the integral**.

   Solution:

   (a)

$$\begin{aligned}
\int_0^1 \int_0^{x^2} xe^{x^2+y} dydx &= \int_0^1 \int_0^{x^2} xe^{x^2} e^y dydx \\
&= \int_0^1 xe^{x^2} e^y \Big|_0^{x^2} dx \quad (2 \text{ pts}) \\
&= \int_0^1 xe^{x^2}(e^{x^2} - 1) dx \\
&= \int_0^1 xe^{2x^2} - xe^{x^2} dx \\
&= \left(\frac{1}{4} e^{2x^2} (1 \text{ pt}) - \frac{1}{2} e^{x^2} (1 \text{ pt}) \right) \Big|_0^1 \\
&= \frac{1}{4} e^2 - \frac{1}{2} e + \frac{1}{4}. \quad (2 \text{ pts})
\end{aligned}$$

   (b)



   (2 pts)

   The iterated integral is

$$\int_0^1 \int_{\sqrt{y}}^1 xe^{x^2+y} dxdy. \quad (2 \text{ pts})$$

2. (10 pts) Let D be the region on x-y plane bounded by $y = 0$, $y = x$ and $x = \sqrt{1 - y^2}$. Evaluate $\iint_D x + 2y \, dA$ using the integral in polar coordinates.

   Solution:

$$D = \{ \, 0 \le \theta \le \frac{\pi}{4}, 0 \le r \le 1 \, \}, \quad (2 \text{ pts})$$

so

$$\begin{aligned}
\iint_D x + 2y \; dA &= \int_0^{\frac{\pi}{4}} \int_0^1 r(\cos\theta + 2\sin\theta) r dr d\theta \quad (2 \text{ pts}) \\
&= \int_0^{\frac{\pi}{4}} (\cos\theta + 2\sin\theta) \frac{1}{3} r^3 \Big|_{r=0}^{r=1} d\theta \quad (2 \text{ pts}) \\
&= \frac{1}{3} \int_0^{\frac{\pi}{4}} \cos\theta + 2\sin\theta \; d\theta \\
&= \frac{1}{3} (\sin\theta (1 \text{ pt}) - 2\cos\theta (1 \text{ pt})) \Big|_0^{\frac{\pi}{4}} \\
&= \frac{1}{3} (2 - \frac{\sqrt{2}}{2}) = \frac{4 - \sqrt{2}}{6}. \quad (2 \text{ pts})
\end{aligned}$$